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FURTHER RESULTS ON SD-PRIME CORDIAL GRAPHS

A. Lourdusamy[1*] and S. Jenifer Wency[2]

[1,2]Department of Mathematics, St. Xavier's College (Autonomous), Palayamkottai - 627 002, Tamil Nadu, India.

[1]E-mail: lourdusamy15@gmail.com
[2]E-mail: jeniferwency@gmail.com

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Abstract. Given a bijection $f : V(G) \to \{1, 2, \cdots, |V(G)|\}$, we associate 2 integers $S = f(u) + f(v)$ and $D = |f(u) - f(v)|$ with every edge uv in $E(G)$. The labeling f induces an edge labeling $f^{'} : E(G) \to \{0, 1\}$ such that for any edge uv in $E(G)$, $f^{'}(uv) = 1$ if $gcd(S, D) = 1$, and $f^{'}(uv) = 0$ otherwise. Let $e_{f'}(i)$ be the number of edges labeled with $i \in \{0, 1\}$. We say f is SD-prime cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Moreover G is SD-prime cordial if it admits SD-prime cordial labeling. In this paper, we prove that $K_2 + mK_1$, $\left\langle K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} \right\rangle$, jelly fish graph, petersen graph, friendship graph if $n \equiv 0, 1, 3 \ (mod\ 4)$ and gear graph are SD-prime cordial graphs.

Keywords: SD-prime labeling, SD-prime cordial labeling.

2010 Mathematics Subject Classification: 05C78

1. INTRODUCTION

Let $G = (V(G), E(G))$ be a simple, finite and undirected graph of order $|V(G)| = p$ and size $|E(G)| = q$. We follow the basic notations and terminology as in [2]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of vertices the labeling is called vertex labeling. If the domain is the set of edges, then we called about edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. For all detailed survey of graph labeling we to refer Gallian [1]. In [4, 5], Lau et al. have introduced the concepts SD-prime labeling. In [3], Lau et al. have introduced SD-prime cordial labeling and they proved behaviour of several graphs like path, complete bipartite graph, star, double star, wheel, fan, double fan and ladder are SD-prime cordial graph. In [7, 8], Lourdusamy et al. proved that the graphs $S^{'}(K_{1,n})$, $D_2(K_{1,n})$, $S(K_{1,n})$, $DS(K_{1,n})$, $S^{'}(B_{n,n})$, $D_2(B_{n,n})$, $DS(B_{n,n})$, $S(B_{n,n})$, $K_{1,3} * K_{1,n}$, CH_n, Fl_n, P_n^2, $T(P_n)$, $T(C_n)$, the graph obtained by duplication of each vertex of path and cycle by an edge, Q_n, $A(T_n)$, TL_n, $P_n \odot K_1$, $C_n \odot K_1$ and J_n admit SD-prime cordial labeling. In [6], we proved that the union of star and path graph, subdivision of comb graph, subdivision of ladder graph and the graph obtained by attaching star graph at one end of the path are SD-prime cordial graphs. We proved that the union of two SD-prime cordial graphs need not be SD-prime cordial graph. Also, given a positive integer n, there is SD-prime cordial graph G with n vertices. In this paper, we prove that $K_2 + mK_1$, $\left\langle K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} \right\rangle$, jelly fish graph, petersen graph, friendship graph if $n \equiv 0, 1, 3 \ (mod\ 4)$ and gear graph are SD-prime cordial graphs.

Given a bijection $f : V(G) \to \{1, 2, \cdots, |V(G)|\}$, we associate 2 integers $S = f(u) + f(v)$ and $D = |f(u) - f(v)|$ with every edge uv in E.

Definition 1.1. [4] *A bijection $f : V(G) \to \{1, 2, \cdots, |V(G)|\}$ induces an edge labeling $f^{'} : E(G) \to \{0, 1\}$ such that for any edge uv in G, $f^{'}(uv) = 1$ if $gcd(S, D) = 1$, and $f^{'}(uv) = 0$ otherwise. We say f is SD-prime labeling if $f^{'}(uv) = 1$ for all $uv \in E(G)$. Moreover, G is SD-prime if it admits SD-prime labeling.*

Definition 1.2. [3] *A bijection $f : V(G) \to \{1, 2, \cdots, |V(G)|\}$ induces an edge labeling $f^{'} : E(G) \to \{0, 1\}$ such that for any edge uv in G, $f^{'}(uv) = 1$ if $gcd(S, D) = 1$, and $f^{'}(uv) = 0$ otherwise. The labeling f is called SD-prime cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. We say that G is SD-prime cordial if it admits SD-prime cordial labeling.*

Definition 1.3. *The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set is $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$.*

Definition 1.4. *Consider two copies of graph G namely G_1 and G_2. Then the graph $G^{'} = \langle G_1 \Delta G_2 \rangle$ is the graph obtained by joining the apex vertices of G_1 and G_2 by an edge as well as to a new vertex x.*

Definition 1.5. *The jelly fish graph $(JF)_n$ is a graph with vertex set $V((JF)_n) = \{u, v, u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and edge set $E((JF)_n) = \{uu_i : 1 \leq i \leq n\} \bigcup \{vv_j : 1 \leq j \leq n-2\} \bigcup \{u_{n-1}u_n, vu_n, vu_{n-1}\}$.*

Definition 1.6. *The friendship graph FR_n is a one point union of n copies of cycle C_3.*

Definition 1.7. *The gear graph G_n is obtained from the wheel by subdividing each of its rim edge.*

2. Main Results

In this section, we prove that the SD-prime cordial labeling of joining of certain graph, $\left\langle K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} \right\rangle$, jelly fish graph, petersen graph, friendship graph if $n \equiv 0, 1, 3 \ (mod\ 4)$ and gear graph.

Theorem 2.1. *The graph $K_2 + mK_1$ admits SD-prime cordial labeling.*

Proof. Let $G = K_2 + mK_1$. Let $V(G) = \{u, v, w_1, w_2, \cdots, w_m\}$ and $E(G) = \{uv, uw_i, vw_i : 1 \le i \le m\}$. Then G is of order $m+2$ and size $2m+1$. Define $f : V(G) \to \{1, 2, \cdots, m+2\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(w_i) = i + 2,\ 1 \le i \le m.$$

Then, the induced edge labels are

$$f'(uv) = 1;$$
$$f'(uw_{2i-1}) = 0,\ 1 \le i \le \left\lceil \frac{m}{2} \right\rceil;$$
$$f'(uw_{2i}) = 1,\ 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor;$$
$$f'(vw_{2i-1}) = 1,\ 1 \le i \le \left\lceil \frac{m}{2} \right\rceil;$$
$$f'(vw_{2i}) = 0,\ 1 \le i \le \left\lfloor \frac{m}{2} \right\rfloor.$$

We observe that, $e_{f'}(0) = m$ and $e_{f'}(1) = m + 1$.
Thus, $|e_{f'}(0) - e_{f'}(1)| \le 1$. Hence, $K_2 + mK_1$ admits SD-prime cordial labeling. □

Theorem 2.2. *The graph $\left\langle K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} \right\rangle$ admits SD-prime cordial labeling.*

Proof. Let $u_1, u_2, \cdots, u_n$ be the pendant vertices of $K_{1,n}^{(1)}$ and $v_1, v_2, \cdots, v_n$ be the pendant vertices of $K_{1,n}^{(2)}$. Let u and v be the apex vertices of $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ respectively and u, v are adjacent to a new common vertex x. Then, $\left\langle K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} \right\rangle$ is of order $2n + 3$ and size $2n + 3$.
Define $f : V\left(\left\langle K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} \right\rangle\right) \to \{1, 2, 3, \cdots, 2n + 3\}$ as follows:

$$f(x) = 3;$$
$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(u_i) = \begin{cases} (2i-1) + 4 & \text{if } i \text{ is odd and } 1 \le i \le n \\ 2(i-1) + 4 & \text{if } i \text{ is even and } 1 \le i \le n; \end{cases}$$
$$f(v_i) = \begin{cases} 2(i-1) + 4 & \text{if } i \text{ is odd and } 1 \le i \le n \\ (2i-1) + 4 & \text{if } i \text{ is even and } 1 \le i \le n. \end{cases}$$

Then, the induced edge labels are

$$f'(xu) = 0;$$
$$f'(xv) = f'(uv) = 1;$$
$$f'(uu_i) = f'(vv_i) = \begin{cases} 0 & \text{if } i \text{ is odd and } 1 \le i \le n \\ 1 & \text{if } i \text{ is even and } 1 \le i \le n. \end{cases}$$

We observe that,

$$e_{f'}(0) = \begin{cases} n+2 & \text{if } n \text{ is odd} \\ n+1 & \text{if } n \text{ is even.} \end{cases}$$

$$e_{f'}(1) = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n+2 & \text{if } n \text{ is even.} \end{cases}$$

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $\left\langle K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} \right\rangle$ admits SD-prime cordial labeling. □

Example 2.3. A SD-prime cordial labeling of $\left\langle K_{1,4}^{(1)} \Delta K_{1,4}^{(2)} \right\rangle$ is shown in Figure 1.

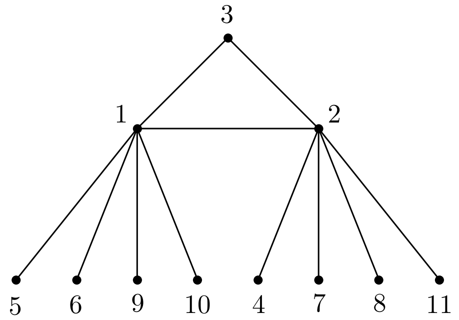

FIGURE 1

Theorem 2.4. *The jelly fish graph $(JF)_n$ admits SD-prime cordial labeling.*

Proof. Let $V((JF)_n) = \{u, v, u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and $E((JF)_n) = \{uu_i : 1 \leq i \leq n\} \bigcup \{vv_j : 1 \leq j \leq n-2\} \bigcup \{u_{n-1}u_n, vu_n, vu_{n-1}\}$. Then $(JF)_n$ is of order $2n$ and size $2n+1$. Define $f : V((JF)_n) \to \{1, 2, \cdots, 2n\}$ as follows:

$f(v) = 1;$

$f(u) = 2;$

$f(u_{n-1}) = 3;$

$f(u_n) = 4;$

$$f(v_j) = \begin{cases} (2j-1)+4 & \text{if } j \text{ is odd and } 1 \leq j \leq n-2 \\ 2j+4 & \text{if } j \text{ is even and } 1 \leq j \leq n-2; \end{cases}$$

$$f(u_i) = \begin{cases} 2i+4 & \text{if } i \text{ is odd and } 1 \leq i \leq n-2 \\ (2i-1)+4 & \text{if } i \text{ is even and } 1 \leq i \leq n-2. \end{cases}$$

Then, the induced edge labels are

$f'(vu_{n-1}) = 0;$

$f'(vu_n) = 1;$

$f'(u_{n-1}u_n) = 1;$

$f'(uu_{n-1}) = 1;$

$f'(uu_n) = 0;$

$$f'(vv_j) = \begin{cases} 0 & \text{if } j \text{ is odd and } 1 \leq j \leq n-2 \\ 1 & \text{if } j \text{ is even and } 1 \leq j \leq n-2; \end{cases}$$
$$f'(uu_i) = \begin{cases} 0 & \text{if } i \text{ is odd and } 1 \leq i \leq n-2 \\ 1 & \text{if } i \text{ is even and } 1 \leq i \leq n-2. \end{cases}$$

We observe that,
$$e_{f'}(0) = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even.} \end{cases}$$
$$e_{f'}(1) = \begin{cases} n & \text{if } n \text{ is odd} \\ n+1 & \text{if } n \text{ is even.} \end{cases}$$

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $(JF)_n$ admits SD-prime cordial labeling. □

Example 2.5. A SD-prime cordial labeling of $(JF)_6$ is shown in Figure 2.

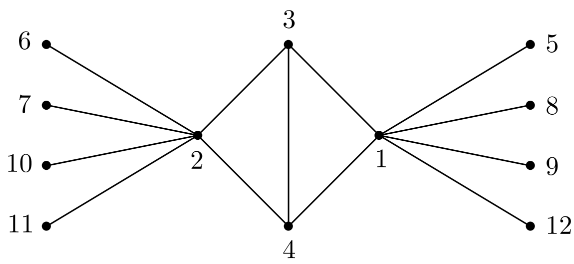

FIGURE 2

Theorem 2.6. *The petersen graph admits SD-prime cordial labeling.*

Proof. Let u_1, u_2, u_3, u_4, u_5 be the internal vertices and let v_1, v_2, v_3, v_4, v_5 be the external vertices of Petersen graph such that each u_i is adjacent to v_i for $1 \leq i \leq 5$. We define a vertex labeling f such that $f(u_i) = i$ for $1 \leq i \leq 5$, $f(v_i) = i + 4$ for $2 \leq i \leq 5$ and $f(v_1) = 10$. Then, the induced edge labels are

$$f'(u_iv_i) = 0, \ 2 \leq i \leq 5;$$
$$f'(u_1v_1) = 1;$$
$$f'(u_1u_2) = 0;$$
$$f'(u_iu_{i+1}) = 1, \ 2 \leq i \leq 4;$$
$$f'(v_1v_3) = f'(v_2v_4) = f'(v_3v_5) = 0;$$
$$f'(v_1v_4) = f'(v_2v_5) = 1.$$

We observe that, $e_{f'}(0) = 8$ and $e_{f'}(1) = 7$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, petersen graph admits SD-prime cordial labeling. □

Example 2.7. A SD-prime cordial labeling of petersen graph is shown in Figure 3.

Theorem 2.8. *The friendship graph FR_n admits SD-prime cordial labeling if $n \equiv 0, 1, 3\ (mod\, 4)$.*

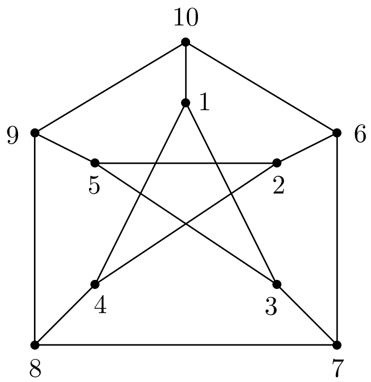

FIGURE 3

Proof. Let $v, v_1, v_2, \cdots, v_{2n}$ be the vertices of FR_n. Then FR_n is of order $2n+1$ and size $3n$. Define $f : V(FR_n) \to \{1, 2, \cdots, 2n+1\}$ as follows:

Cases 1. $n \equiv 1\ (mod\ 4)$.

For first $\frac{n+1}{2}$ copies of C_3, we assign the labels $1, 2, 3, 4, \cdots, n+2$ to the vertices $v, v_1, v_2, \cdots, v_{n+1}$. For next $\frac{n-1}{4}$ copies of C_3, we assign the labels $n+3, n+5, \cdots, 2n$ to the vertices $v_{n+2}, v_{n+3}, \cdots, v_{\frac{3n+1}{2}}$. Remaining $\frac{n-1}{4}$ copies of C_3, we assign the labels $n+4, n+6, \cdots, 2n+1$ to the vertices $v_{\frac{3n+1}{2}+1}, v_{\frac{3n+1}{2}+2}, \cdots, v_{2n}$. We observe that, $e_{f'}(0) = \frac{3n-1}{2}$ and $e_{f'}(1) = \frac{3n+1}{2}$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, f admits SD-prime cordial labeling.

Cases 2. $n \equiv 3\ (mod\ 4)$.

For first $\frac{n-1}{2}$ copies of C_3, we assign the labels $1, 2, 3, 4, \cdots, n$ to the vertices $v, v_1, v_2, \cdots, v_{n-1}$. For next $\frac{n+1}{4}$ copies of C_3, we assign the labels $n+1, n+3, \cdots, 2n$ to the vertices $v_n, v_{n+1}, \cdots, v_{\frac{3n-1}{2}}$. Remaining $\frac{n+1}{4}$ copies of C_3, we assign the labels $n+2, n+4, \cdots, 2n+1$ to the vertices $v_{\frac{3n-1}{2}+1}, v_{\frac{3n-1}{2}+2}, \cdots, v_{2n}$. We observe that, $e_{f'}(0) = \frac{3n+1}{2}$ and $e_{f'}(1) = \frac{3n-1}{2}$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, f admits SD-prime cordial labeling.

Cases 3. $n \equiv 0\ (mod\ 4)$.

For first $\frac{n}{2}$ copies of C_3, we assign the labels $1, 2, 3, 4, \cdots, n+1$ to the vertices $v, v_1, v_2, \cdots, v_n$. For next $\frac{n}{4}$ copies of C_3, we assign the labels $n+2, n+4, \cdots, 2n$ to the vertices $v_{n+1}, v_{n+2}, \cdots, v_{\frac{3n}{2}}$. Remaining $\frac{n}{4}$ copies of C_3, we assign the labels $n+3, n+5, \cdots, 2n+1$ to the vertices $v_{\frac{3n}{2}+1}, v_{\frac{3n}{2}+2}, \cdots, v_{2n}$. We observe that, $e_{f'}(0) = \frac{3n}{2}$ and $e_{f'}(1) = \frac{3n}{2}$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, f admits SD-prime cordial labeling.

□

Theorem 2.9. *The gear graph G_n admits SD-prime cordial labeling.*

Proof. Let W_n be the wheel with apex vertex v and rim vertices $v_1, v_2, \cdots, v_n$. To obtain the gear graph G_n subdivide each rim edge of wheel by the vertices $u_1, u_2, \cdots, u_n$. Where each u_i is added between v_i and v_{i+1} for $i = 1, 2, \cdots, n-1$ and u_n is added between v_1 and v_n. Then G_n is of order $2n+1$ and size $3n$. Define $f : V(G_n) \to \{1, 2, 3, \cdots, 2n+1\}$ as follows:

Case 1. n is odd.

$$f(v) = 2;$$
$$f(v_i) = \begin{cases} i & \text{if } i \equiv 1\ (mod\ 2) \text{ and } 1 \leq i \leq n \\ i+1 & \text{if } i \equiv 0\ (mod\ 2) \text{ and } 1 \leq i \leq n; \end{cases}$$
$$f(u_i) = \begin{cases} i+1 & \text{if } i \equiv 1\ (mod\ 2) \text{ and } 1 \leq i \leq n \\ i+2 & \text{if } i \equiv 0\ (mod\ 2) \text{ and } 1 \leq i \leq n. \end{cases}$$

In view of the above labeling, $e_{f'}(0) = \left\lceil \frac{3n}{2} \right\rceil$ and $e_{f'}(1) = \left\lfloor \frac{3n}{2} \right\rfloor$.

Case 2. n is even.

Assign the labels to the vertices v, v_i for $1 \leq i \leq n$, u_i for $1 \leq i \leq n-1$ as in Case 1 and $f(u_n) = 2n+1$. In view of the above labeling, $e_{f'}(0) = \frac{3n}{2}$ and $e_{f'}(1) = \frac{3n}{2}$.

Thus in the above two case, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, f admits SD-prime cordial labeling. □

3. CONCLUSION

We have proved that the joining of certain graph, $\left\langle K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} \right\rangle$, jelly fish graph, petersen graph, friendship graph if $n \equiv 0, 1, 3\ (mod\ 4)$ and gear graph are SD-prime cordial graphs.

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